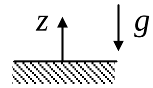
ME 309 Formula Sheet

Basic Equations

$p = \rho RT$

$$\tau_{yx} = \mu \frac{du_x}{dy} \qquad F = \int dF = \int_A \tau \, dA \qquad \frac{dp}{dz} = -\rho g$$



$$\frac{D}{Dt}\int_{V_{\text{system}}} \eta\rho dV = \frac{d}{dt}\int_{\text{CV}} \eta\rho dV + \int_{\text{CS}} \eta\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)$$

$$0 = \frac{d}{dt}\int_{\text{CV}} \rho dV + \int_{\text{CS}} \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)$$

$$\mathbf{F}_{\text{S}} + \mathbf{F}_{\text{B}} = \frac{d}{dt}\int_{\text{CV}} \mathbf{u}_{XYZ}\rho dV + \int_{\text{CS}} \mathbf{u}_{XYZ}\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)$$

$$\mathbf{F}_{\text{S}} + \mathbf{F}_{\text{B}} - \int_{\text{CV}} \mathbf{a}_{xyz/XYZ}\rho dV = \frac{d}{dt}\int_{\text{CV}} \mathbf{u}_{xyz}\rho dV + \int_{\text{CS}} \mathbf{u}_{xyz}\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)$$

$$\mathbf{M}_{\text{S}} + \mathbf{M}_{\text{B}} = \frac{d}{dt}\int_{\text{CV}} \left(\mathbf{r}\times\mathbf{u}\right)\rho dV + \int_{\text{CS}} \left(\mathbf{r}\times\mathbf{u}\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)$$

$$\dot{Q}_{\text{into CV}} + \dot{W}_{\text{on CV}} = \frac{d}{dt}\int_{\text{CV}} e\rho dV + \int_{\text{CS}} \left(e + p/\rho\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) \text{ where } e = u + \frac{V^2}{2} + gz$$

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant} \quad \text{(for inviscid flow)}$$

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{lT} - h_s = f\,\frac{L}{D}\,\frac{\bar{V}^2}{2} + \Sigma K \frac{\bar{V}^2}{2} - h_s$$

$$\left(\frac{p}{\rho g} + \alpha \frac{\bar{V}^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha \frac{\bar{V}^2}{2g} + z\right)_1 - H_L + H_S \quad \text{where } H_L = f\,\frac{L}{D}\,\frac{\bar{V}^2}{2g} + \Sigma K \frac{\bar{V}^2}{2g} \text{ and } H_S = \frac{\dot{W}_S}{\dot{m} g}$$

$$T_{\text{shaft}} = (r_2 V_{t2} - r_1 V_{t1})\dot{m}; \quad H_p = \left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_{\text{discharge}} - \left[\frac{p}{\rho g} + \frac{V^2}{2g} + z\right]_{\text{suction}}; \quad \text{NPSH} = \frac{p_s}{\rho g} + \frac{V_s^2}{2g} - \frac{p_v}{\rho g}$$

$$\delta_D = \delta^* = \int_0^\delta (1 - \frac{u}{U})dy; \quad \delta_M = \theta = \int_0^\delta \frac{u}{U}(1 - \frac{u}{U})dy; \quad \tau_w = \rho U^2 \frac{\partial}{\partial x}\int_0^\delta \frac{u}{U}(1 - \frac{u}{U})dy = \rho U^2 \frac{d\theta}{dx}$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}; \qquad C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}; \qquad \frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx}$$

$$u_R = \pm\left[\left(\frac{x_1}{R}\,\frac{\partial R}{\partial x_1} u_1\right)^2 + \left(\frac{x_2}{R}\,\frac{\partial R}{\partial x_2} u_2\right)^2 + \cdots + \left(\frac{x_n}{R}\,\frac{\partial R}{\partial x_n} u_n\right)^2\right]^{1/2}$$

Data

$$\left.\begin{aligned}\rho &= 1000\ \text{kg/m}^3\\ \mu &= 1.00\times10^{-3}\ \text{kg/(m}\cdot\text{s)}\end{aligned}\right\}\text{Water}$$

$$\left.\begin{aligned}\rho &= 1.23\ \text{kg/m}^3\\ \mu &= 1.79\times10^{-5}\ \text{kg/(m}\cdot\text{s)}\end{aligned}\right\}\text{Air (STP)}$$

$$\left.\begin{aligned}c_p &= 1.00\ \text{kJ/(kg}\cdot\text{K)}\\ c_v &= 0.717\ \text{kJ/(kg}\cdot\text{K)}\\ R &= 0.287\ \text{kJ/(kg}\cdot\text{K)}\end{aligned}\right\}\text{Air}$$

Definitions and Conversions

Specific gravity, SG = ρ/ρ_{H_2O} (at 4 ^{o}C)

Kinematic viscosity, $\nu \equiv \mu/\rho$

1 Pa = N/m^2; 1 atm = 101 kPa = 14.7 psia

1 ft = 0.305 m; g = 9.81 m/s^2 = 32.2 ft/s^2

1 lb$_m$ = 0.454 kg; 1 slug = 32.2 lb$_m$

1 lb$_f$ = 32.2 lb$_m$·ft/s^2 = 1 slug·ft/s^2

1 ft^3 = 7.48 gal; 1 m^3 = 10^3 litre

1 hp = 550 ft·lb$_f$/s = 746 W

1 Btu = 778 ft·lb$_f$ = 1.06 kJ

1000 L = 1 m^3

Euler's Equations in Streamline Coordinates

$$\rho\left[\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial s}\right]=-\frac{\partial p}{\partial s}-\rho g\frac{\partial z}{\partial s}$$

$$\rho\frac{V^2}{R}=-\frac{\partial p}{\partial n}+\rho g\frac{\partial z}{\partial n}$$

Parameter	**Laminar (Re < 500,000)**	**Turbulent (Re > 500,000) (Estimated)**
99% thickness, δ	$\frac{\delta}{x}=\frac{5.0}{\text{Re}_x^{1/2}}$	$\frac{\delta}{x}=\frac{0.382}{\text{Re}_x^{1/5}}$
displacement thickness, δ_D or δ^*	$\frac{\delta_D}{x}=\frac{1.72}{\text{Re}_x^{1/2}}$	$\frac{\delta_D}{x}=\frac{0.0478}{\text{Re}_x^{1/5}}$
momentum thickness, δ_M or Θ	$\frac{\delta_M}{x}=\frac{0.664}{\text{Re}_x^{1/2}}$	$\frac{\delta_M}{x}=\frac{0.0371}{\text{Re}_x^{1/5}}$
friction coefficient, C_f	$C_f=\frac{\tau_w}{\frac{1}{2}\rho U^2}=\frac{0.664}{\text{Re}_x^{1/2}}$	$C_f=\frac{\tau_w}{\frac{1}{2}\rho U^2}=\frac{0.0594}{\text{Re}_x^{1/5}}$
drag coefficient, C_D	$C_D=\frac{D}{\frac{1}{2}\rho U^2 LW}=\frac{1.328}{\text{Re}_L^{1/2}}$	$C_D=\frac{D}{\frac{1}{2}\rho U^2 LW}=\frac{0.0742}{\text{Re}_L^{1/5}}$

continuity equation

rectangular coordinates (x, y, z)	cylindrical coordinates (r, θ, z)
$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} = 0$	$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$

stress tensor components for a Newtonian fluid

rectangular coordinates (x, y, z)	cylindrical coordinates (r, θ, z)
$\sigma_{xx} = -p + \mu\left[2\frac{\partial u_x}{\partial x} - \frac{2}{3}(\nabla \cdot \mathbf{u})\right]$	$\sigma_{rr} = -p + \mu\left[2\frac{\partial u_r}{\partial r} - \frac{2}{3}(\nabla \cdot \mathbf{u})\right]$
$\sigma_{yy} = -p + \mu\left[2\frac{\partial u_y}{\partial y} - \frac{2}{3}(\nabla \cdot \mathbf{u})\right]$	$\sigma_{\theta\theta} = -p + \mu\left[2\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right) - \frac{2}{3}(\nabla \cdot \mathbf{u})\right]$
$\sigma_{zz} = -p + \mu\left[2\frac{\partial u_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{u})\right]$	$\sigma_{zz} = -p + \mu\left[2\frac{\partial u_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{u})\right]$
$\sigma_{xy} = \sigma_{yx} = \mu\left[\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right]$	$\sigma_{r\theta} = \sigma_{yx} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right]$
$\sigma_{xz} = \sigma_{zx} = \mu\left[\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right]$	$\sigma_{\theta z} = \sigma_{z\theta} = \mu\left[\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right]$
$\sigma_{yz} = \sigma_{zy} = \mu\left[\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right]$	$\sigma_{zr} = \sigma_{rz} = \mu\left[\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right]$
$\nabla \cdot \mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$	$\nabla \cdot \mathbf{u} = \frac{1}{r}\frac{\partial(r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}$

Navier-Stokes equations for a Newtonian fluid with constant density (ρ) and dynamic viscosity (μ)

rectangular coordinates (x, y, z):

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right] + \rho f_x$$

$$\rho\left(\frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right] + \rho f_y$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho f_z$$

cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_r)\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right] + \rho f_r$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right] + \rho f_\theta$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho f_z$$

Math Formulas

$\frac{d}{dx}(\cos x) = -\sin x$ $\quad \frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\quad \frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$ $\quad \frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\quad \frac{d}{dx}\left[\exp(ax)\right] = a\exp(ax)$

$\int \sin^2 x dx = \tfrac{1}{2}x - \tfrac{1}{4}\sin 2x$ $\quad \int \cos^2 x dx = \tfrac{1}{2}x + \tfrac{1}{4}\sin 2x$

$\int \tan^2 x dx = \tan x - x$ $\quad \int \sin^3 x dx = -\tfrac{1}{3}\left(2 + \sin^2 x\right)\cos x$

$\int \cos^3 x dx = \tfrac{1}{3}\left(2 + \cos^2 x\right)\sin x$ $\quad \int \tan^3 x dx = \tfrac{1}{2}\tan^2 x + \ln\left|\cos x\right|$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

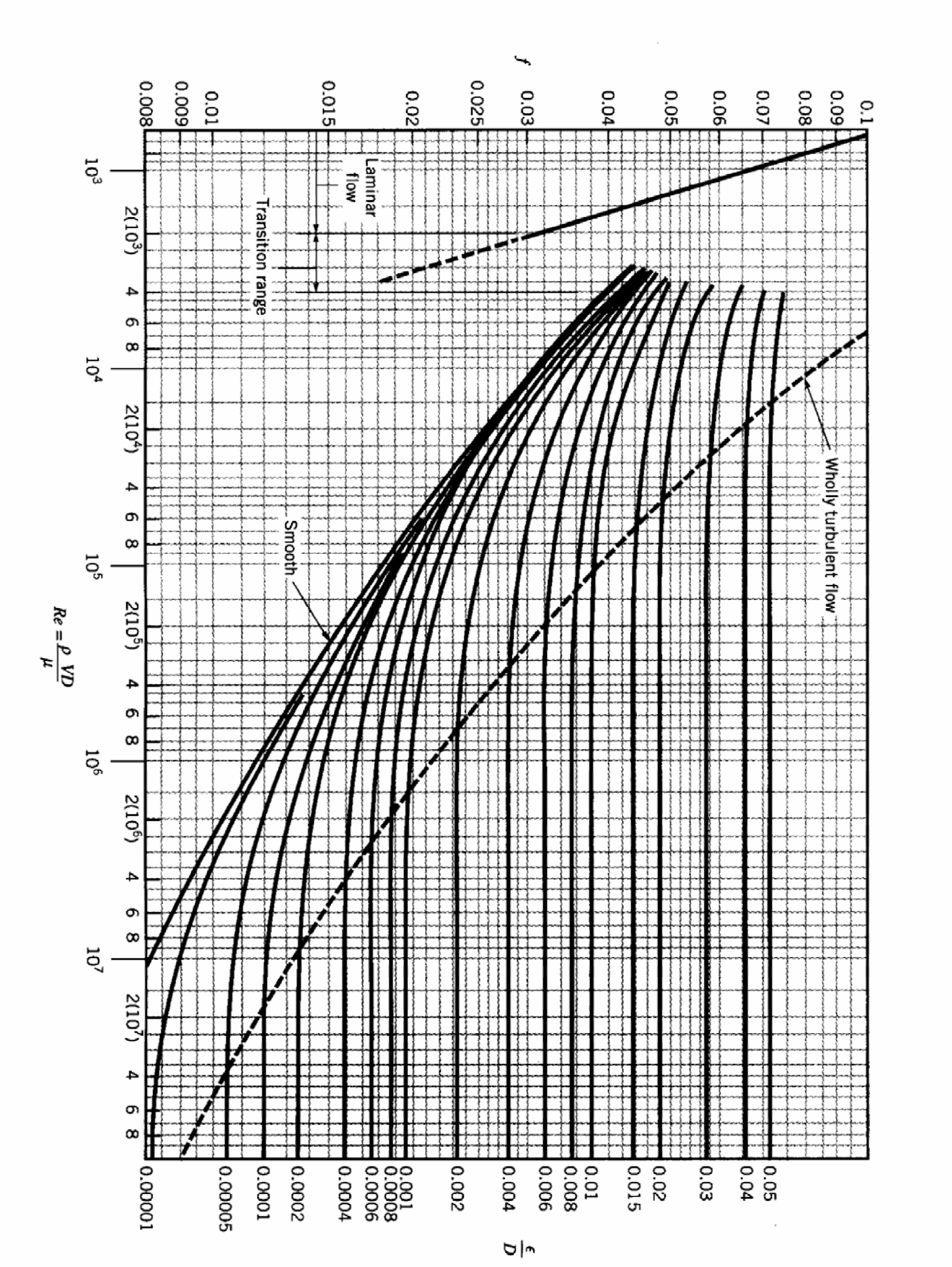
$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(1 - \cos\theta) \quad \cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(1 + \cos\theta)$

miscellaneous vector operations (In the table below: N is a scalar and **n** is a vector.)

rectangular coordinates (x, y, z)	cylindrical coordinates (r, θ, z)
$\nabla N = \frac{\partial N}{\partial x}\hat{\mathbf{e}}_x + \frac{\partial N}{\partial y}\hat{\mathbf{e}}_y + \frac{\partial N}{\partial z}\hat{\mathbf{e}}_z$	$\nabla N = \frac{\partial N}{\partial r}\hat{\mathbf{e}}_r + \frac{1}{r}\frac{\partial N}{\partial \theta}\hat{\mathbf{e}}_\theta + \frac{\partial N}{\partial z}\hat{\mathbf{e}}_z$
$\nabla \cdot \mathbf{n} = \frac{\partial n_x}{\partial x} + \frac{\partial n_y}{\partial y} + \frac{\partial n_z}{\partial z}$	$\nabla \cdot \mathbf{n} = \frac{1}{r}\frac{\partial}{\partial r}(r n_r) + \frac{1}{r}\frac{\partial n_\theta}{\partial \theta} + \frac{\partial n_z}{\partial z}$
$\nabla \times \mathbf{n} = \left(\frac{\partial n_z}{\partial y} - \frac{\partial n_y}{\partial z}\right)\hat{\mathbf{e}}_x + \left(\frac{\partial n_x}{\partial z} - \frac{\partial n_z}{\partial x}\right)\hat{\mathbf{e}}_y + \left(\frac{\partial n_y}{\partial x} - \frac{\partial n_x}{\partial y}\right)\hat{\mathbf{e}}_z$	$\nabla \times \mathbf{n} = \left(\frac{1}{r}\frac{\partial n_z}{\partial \theta} - \frac{\partial n_\theta}{\partial z}\right)\hat{\mathbf{e}}_r + \left(\frac{\partial n_r}{\partial z} - \frac{\partial n_z}{\partial r}\right)\hat{\mathbf{e}}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(r n_\theta) - \frac{\partial n_r}{\partial \theta}\right)\hat{\mathbf{e}}_z$
$\nabla^2 N = \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} + \frac{\partial^2 N}{\partial z^2}$	$\nabla^2 N = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 N}{\partial \theta^2} + \frac{\partial^2 N}{\partial z^2}$
$\frac{DN}{DT} = \frac{\partial N}{\partial t} + u_x\frac{\partial N}{\partial x} + u_y\frac{\partial N}{\partial y} + u_z\frac{\partial N}{\partial z}$	$\frac{DN}{Dt} = \frac{\partial N}{\partial t} + u_r\frac{\partial N}{\partial r} + \frac{u_\theta}{r}\frac{\partial N}{\partial \theta} + u_z\frac{\partial N}{\partial z}$

Lagrangian (aka material, substantial) Acceleration

rectangular coordinates (x, y, z)	cylindrical coordinates (r, θ, z)
$\frac{Du_x}{Dt} = \frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}$ $\frac{Du_y}{Dt} = \frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z}$ $\frac{Du_z}{Dt} = \frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z}$	$\frac{Du_r}{Dt} = \frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}$ $\frac{Du_\theta}{Dt} = \frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}$ $\frac{Du_z}{Dt} = \frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}$


f
Laminar flow
Transition range
Wholly turbulent flow
Smooth
$Re = \frac{\rho VD}{\mu}$
$\frac{\varepsilon}{D}$

Average Roughness, ε, of Commercial Pipes

Material (new)	ft	mm
Riveted steel	0.003-0.03	0.9-9.0
Concrete	0.001-0.01	0.3-3.0
Wood stave	0.0006-0.003	0.18-0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

Table of Minor Loss Coefficients

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bends, flanged	0.2
180° return bends, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.06

Component	K_L
e. Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, ¼ closed	0.26
Gate, ½ closed	2.1
Gate, ¾ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, 1/3 closed	5.5
Ball valve, 2/3 closed	210
f. Entrances	
Re-entrant	0.8
Sharp-edged	0.5
Slightly rounded	0.2
Well rounded	0.04
g. Exits	
Re-entrant, sharp-edged, slightly rounded, well-rounded	1.0

h. Sudden Contraction/Expansion:

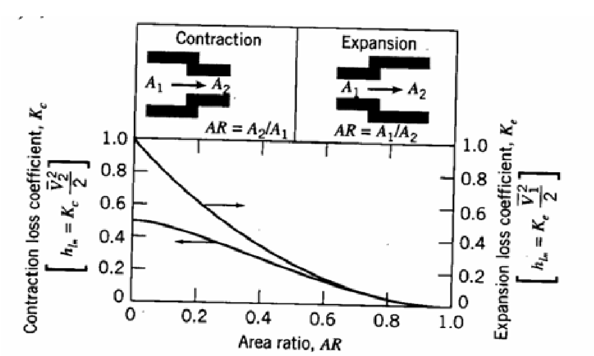


Fig. 8.15 Loss coefficients for flow through sudden area changes. (Data from [1].)

ideal gas relations

$$p = \rho RT \qquad du = c_v dT \qquad dh = c_p dT$$

$$k = c_p / c_v \qquad c_p = c_v + R \qquad c_p = \frac{k}{k-1}R \qquad c_v = \frac{1}{k-1}R$$

$$c = \sqrt{kRT} \qquad ds = c_p \frac{dT}{T} - R\frac{dp}{p} = c_v \frac{dT}{T} + R\frac{dv}{v}$$

properties of air (treated as a perfect gas)

$$k = 1.4 \qquad R = 287 \ \tfrac{\text{N·m}}{\text{kg·K}} \qquad c_p = 1005 \ \tfrac{\text{N·m}}{\text{kg·K}} \qquad c_v = 718 \ \tfrac{\text{N·m}}{\text{kg·K}}$$

$$R = 53.3 \ \tfrac{\text{ft·lb}_\text{f}}{\text{lb}_\text{m}\text{·°R}} \qquad c_p = 187 \ \tfrac{\text{ft·lb}_\text{f}}{\text{lb}_\text{m}\text{·°R}} \qquad c_v = 133 \ \tfrac{\text{ft·lb}_\text{f}}{\text{lb}_\text{m}\text{·°R}}$$

adiabatic relations for a perfect gas

$$T_0 = T + \frac{V^2}{2c_p} \qquad \frac{T}{T_0} = \left(\frac{c}{c_0}\right)^2 = \left(1 + \frac{k-1}{2}\text{Ma}^2\right)^{-1}$$

isentropic relations for a perfect gas

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/k-1} \qquad \frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/k-1} \qquad \frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{k}$$

$$\frac{p}{p_0} = \left(1 + \frac{k-1}{2}\text{Ma}^2\right)^{k/1-k} \qquad \frac{\rho}{\rho_0} = \left(1 + \frac{k-1}{2}\text{Ma}^2\right)^{1/1-k}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}}\left(\frac{1 + \frac{k-1}{2}\text{Ma}^2}{1 + \frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}} \qquad \dot{m}_{\text{choked}} = \left(1 + \frac{k-1}{2}\right)^{\frac{k+1}{2(1-k)}} p_0 \sqrt{\frac{k}{RT_0}}A^*$$

conditions across a normal shock wave

$$\text{Ma}_2^2 = \frac{(k-1)\text{Ma}_1^2 + 2}{2k\text{Ma}_1^2 - (k-1)}$$

$$\frac{p_2}{p_1} = \frac{2k}{k+1}\text{Ma}_1^2 - \frac{k-1}{k+1}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(k+1)\text{Ma}_1^2}{2 + (k-1)\text{Ma}_1^2}$$

$$\frac{T_2}{T_1} = \left[2 + (k-1)\text{Ma}_1^2\right]\frac{2k\text{Ma}_1^2 - (k-1)}{\left[(k+1)\text{Ma}_1\right]^2}$$

$$\frac{p_{02}}{p_{01}} = \frac{\rho_{02}}{\rho_{01}} = \frac{A_1^*}{A_2^*} = \left[\frac{(k+1)\text{Ma}_1^2}{2 + (k-1)\text{Ma}_1^2}\right]^{k/(k-1)}\left[\frac{k+1}{2k\text{Ma}_1^2 - (k-1)}\right]^{1/(k-1)}$$

$$\frac{T_{02}}{T_{01}} = 1$$